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JOHN BIRD

SEVENTH EDITION

ROUTLEDGE

Engineering Mathematics

Why is knowledge of mathematics important in engineering?

A career in any engineering or scientific field will require both basic and advanced mathematics. Without mathematics to determine principles, calculate dimensions and limits, explore variations, prove concepts, and so on, there would be no mobile telephones, televisions, stereo systems, video games, microwave ovens, computers, or virtually anything electronic. There would be no bridges, tunnels, roads, skyscrapers, automobiles, ships, planes, rockets or most things mechanical. There would be no metals beyond the common ones, such as iron and copper, no plastics, no synthetics. In fact, society would most certainly be less advanced without the use of mathematics throughout the centuries and into the future.

Electrical engineers require mathematics to design, develop, test, or supervise the manufacturing and installation of electrical equipment, components, or systems for commercial, industrial, military, or scientific use.

Mechanical engineers require mathematics to perform engineering duties in planning and designing tools, engines, machines, and other mechanically functioning equipment; they oversee installation, operation, maintenance, and repair of such equipment as centralised heat, gas, water, and steam systems.

Aerospace engineers require mathematics to perform a variety of engineering work in designing, constructing, and testing aircraft, missiles, and spacecraft; they conduct basic and applied research to evaluate adaptability of materials and equipment to aircraft design and manufacture and recommend improvements in testing equipment and techniques.

Nuclear engineers require mathematics to conduct research on nuclear engineering problems or apply principles and theory of nuclear science to problems concerned with release, control, and utilisation of nuclear energy and nuclear waste disposal.

Petroleum engineers require mathematics to devise methods to improve oil and gas well production and determine the need for new or modified tool designs; they oversee drilling and offer technical advice to achieve economical and satisfactory progress.

Industrial engineers require mathematics to design, develop, test, and evaluate integrated systems for managing industrial production processes, including human work factors, quality control, inventory control, logistics and material flow, cost analysis, and production co-ordination.

Environmental engineers require mathematics to design, plan, or perform engineering duties in the prevention, control, and remediation of environmental health hazards, using various engineering disciplines; their work may include waste treatment, site remediation, or pollution control technology.

Civil engineers require mathematics at all levels in civil engineering – structural engineering, hydraulics and geotechnical engineering are all fields that employ mathematical tools such as differential equations, tensor analysis, field theory, numerical methods and operations research.

Knowledge of mathematics is therefore needed by each of the engineering disciplines listed above.

It is intended that this text – *Engineering Mathematics* – will provide a step-by-step approach to learning fundamental mathematics needed for your engineering studies.

In memory of Elizabeth

Engineering Mathematics

Seventh Edition

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Contents

Preface

'Engineering Mathematics 7th Edition' covers a wide range of syllabus requirements. In particular, the book is suitable for any course involving engineering mathematics and in particular for the latest **National Certificate and Diploma courses and City & Guilds syllabuses in Engineering**.

This text will provide a foundation in mathematical principles, which will enable students to solve mathematical, scientific and associated engineering principles. In addition, the material will provide engineering applications and mathematical principles necessary for advancement onto a range of Incorporated Engineer degree profiles. It is widely recognised that a students' ability to use mathematics is a key element in determining subsequent success. First year undergraduates who need some remedial mathematics will also find this book meets their needs.

In *Engineering Mathematics 7th Edition*, **new material** is included on points of inflexion and Gaussian elimination; in addition, three chapters found on the internet in the previous edition – linear correlation, linear regression and sampling and estimation theories – have been added to the text. Another new feature is a list of essential formulae at the end of the book.

Throughout the text, theory is introduced in each chapter by a simple outline of essential definitions, formulae, laws and procedures. The theory is kept to a minimum, for **problem solving** is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.

For clarity, the text is divided into **eleven topic areas**, these being: number and algebra, areas and volumes, trigonometry, graphs, complex numbers, vectors, statistics, differential calculus, integral calculus, further number and algebra and differential equations.

This new edition covers, in particular, the following syllabuses:

(i) **Mathematics for Technicians**, the core unit for **National Certificate/Diploma** courses in Engineering, to include all or part of the following chapters:
   1. **Algebraic methods**: 2, 5, 11, 13, 14, 28, 30 (1, 4, 8, 9 and 10 for revision)
   2. **Trigonometric methods and areas and volumes**: 18–20, 22–25, 33, 34
   3. **Statistical methods**: 37, 38
   4. **Elementary calculus**: 45, 51, 58

(ii) **Further Mathematics for Technicians**, the optional unit for **National Certificate/Diploma** courses in Engineering, to include all or part of the following chapters:
   1. **Advanced graphical techniques**: 29–31
   2. **Algebraic techniques**: 15, 33, 37, 38
   3. **Trigonometry**: 22–27
   4. **Calculus**: 45–47, 51, 57–59

(iii) **Mathematics contents of City & Guilds Technician Certificate/Diploma courses**

(iv) Any **introductory/access/foundation course** involving Engineering Mathematics **at University, Colleges of Further and Higher education and in schools**.

Each topic considered in the text is presented in a way that assumes in the reader little previous knowledge of that topic.

'Engineering Mathematics 7th Edition' provides a follow-up to '***Basic Engineering Mathematics 6th Edition'*** and a lead into '***Higher Engineering Mathematics 7th Edition'***.

This textbook contains over **1000 worked problems**, followed by some **1800 further problems** (all **with answers at the back of the book**). The further problems are contained within some **237 Practice Exercises**; each Exercise follows on directly from the relevant section of work, every two or three pages. In addition, the text contains **238 multiple-choice questions**. Where at all possible, the problems mirror practical situations found in engineering and science. **525 line diagrams** enhance the understanding of the theory.

At regular intervals throughout the text are some **19 Revision Tests** to check understanding. For example, Revision Test 1 covers material contained in Chapters 1 to 4, Revision Test 2 covers the material in Chapters 5 to 8, and so on. These Revision Tests do not have answers given since it is envisaged that lecturers could set the tests for students to attempt as part of their course structure. Lecturers' may obtain a set of solutions of the Revision Tests in an **Instructor's Manual** available via the internet – see below.

A list of **Essential Formulae** is included in the text for convenience of reference.

'**Learning by Example'** is at the heart of **'*Engineering Mathematics 7th Edition*'**.

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Free Web downloads

For students

1. **Full solutions** to the 1800 questions contained in the 237 Practice Exercises
2. Download **Multiple choice questions and answer sheet**
3. **List of Essential Formulae**
4. **Famous Engineers/Scientists** – 24 are mentioned in the text.

For instructors/lecturers

1. **Full solutions** to the 1800 questions contained in the 237 Practice Exercises
2. **Full solutions** and marking scheme to each of the **19 Revision Tests** – named as **Instructors guide**
3. **Revision Tests – available to run off to be given to students**
4. **Download Multiple choice questions and answer sheet**
5. **List of Essential Formulae**
6. **Illustrations – all 525 available on PowerPoint**
7. **Famous Engineers/Scientists** – 24 are mentioned in the text

Section 1

Number and algebra

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Chapter 1

Revision of fractions, decimals and percentages

Why it is important to understand: **Revision of fractions, decimals and percentages**

Engineers use fractions all the time, examples including stress to strain ratios in mechanical engineering, chemical concentration ratios and reaction rates, and ratios in electrical equations to solve for current and voltage. Fractions are also used everywhere in science, from radioactive decay rates to statistical analysis. Also, engineers and scientists use decimal numbers all the time in calculations. Calculators are able to handle calculations with fractions and decimals; however, there will be times when a quick calculation involving addition, subtraction, multiplication and division of fractions and decimals is needed. Engineers and scientists also use percentages a lot in calculations; for example, percentage change is commonly used in engineering, statistics, physics, finance, chemistry, and economics. When you feel able to do calculations with basic arithmetic, fractions, decimals and percentages, with or without the aid of a calculator, then suddenly mathematics doesn't seem quite so difficult.

At the end of this chapter, you should be able to:

- add, subtract, multiply and divide with fractions
- understand practical examples involving ratio and proportion
- add, subtract, multiply and divide with decimals
- understand and use percentages

1.1 Fractions

When 2 is divided by 3, it may be written as $\frac{2}{3}$ or 2/3. $\frac{2}{3}$ is called a **fraction**. The number above the line, i.e. 2, is called the **numerator** and the number below the line, i.e. 3, is called the **denominator**.

When the value of the numerator is less than the value of the denominator, the fraction is called a **proper fraction**; thus $\frac{2}{3}$ is a proper fraction. When the value of the numerator is greater than the denominator, the fraction is called an **improper fraction**. Thus $\frac{7}{3}$ is an improper fraction and can also be expressed as a **mixed number**, that is, an integer and a proper fraction. Thus the improper fraction $\frac{7}{3}$ is equal to the mixed number $2\frac{1}{3}$

When a fraction is simplified by dividing the numerator and denominator by the same number, the process is called **cancelling**. Cancelling by 0 is not permissible.

Problem 1. Simplify: $\frac{1}{3}+\frac{2}{7}$

The lowest common multiple (i.e. LCM) of the two denominators is 3×7, i.e. 21

Expressing each fraction so that their denominators are 21, gives:

$$\frac{1}{3}+\frac{2}{7}=\frac{1}{3}\times\frac{7}{7}+\frac{2}{7}\times\frac{3}{3}=\frac{7}{21}+\frac{6}{21}$$
$$=\frac{7+6}{21}=\mathbf{\frac{13}{21}}$$

Alternatively:

$$\frac{1}{3}+\frac{2}{7}=\frac{\overset{\text{Step (2)}}{\overset{\downarrow}{(7\times 1)}}+\overset{\text{Step (3)}}{\overset{\downarrow}{(3\times 2)}}}{\underset{\text{Step (1)}}{\underset{\uparrow}{21}}}$$

Step 1: the LCM of the two denominators;

Step 2: for the fraction $\frac{1}{3}$, 3 into 21 goes 7 times, $7\times$ the numerator is 7×1;

Step 3: for the fraction $\frac{2}{7}$, 7 into 21 goes 3 times, $3\times$ the numerator is 3×2

Thus $\frac{1}{3}+\frac{2}{7}=\frac{7+6}{21}=\mathbf{\frac{13}{21}}$ as obtained previously.

Problem 2. Find the value of $3\frac{2}{3}-2\frac{1}{6}$

One method is to split the mixed numbers into integers and their fractional parts. Then

$$3\frac{2}{3}-2\frac{1}{6}=\left(3+\frac{2}{3}\right)-\left(2+\frac{1}{6}\right)$$
$$=3+\frac{2}{3}-2-\frac{1}{6}$$
$$=1+\frac{4}{6}-\frac{1}{6}=1\frac{3}{6}=\mathbf{1\frac{1}{2}}$$

Another method is to express the mixed numbers as improper fractions.

Since $3=\frac{9}{3}$, then $3\frac{2}{3}=\frac{9}{3}+\frac{2}{3}=\frac{11}{3}$

Similarly, $2\frac{1}{6}=\frac{12}{6}+\frac{1}{6}=\frac{13}{6}$

Thus $3\frac{2}{3}-2\frac{1}{6}=\frac{11}{3}-\frac{13}{6}=\frac{22}{6}-\frac{13}{6}=\frac{9}{6}=\mathbf{1\frac{1}{2}}$ as obtained previously.

Problem 3. Determine the value of

$$4\frac{5}{8}-3\frac{1}{4}+1\frac{2}{5}$$

$$4\frac{5}{8}-3\frac{1}{4}+1\frac{2}{5}=(4-3+1)+\left(\frac{5}{8}-\frac{1}{4}+\frac{2}{5}\right)$$
$$=2+\frac{5\times 5-10\times 1+8\times 2}{40}$$
$$=2+\frac{25-10+16}{40}$$
$$=2+\frac{31}{40}=\mathbf{2\frac{31}{40}}$$

Problem 4. Find the value of $\frac{3}{7}\times\frac{14}{15}$

Dividing numerator and denominator by 3 gives:

$$\frac{{}^{1}\cancel{3}}{7}\times\frac{14}{\cancel{15}_{5}}=\frac{1}{7}\times\frac{14}{5}=\frac{1\times 14}{7\times 5}$$

Dividing numerator and denominator by 7 gives:

$$\frac{1\times\cancel{14}^{2}}{{}_{1}\cancel{7}\times 5}=\frac{1\times 2}{1\times 5}=\mathbf{\frac{2}{5}}$$

This process of dividing both the numerator and denominator of a fraction by the same factor(s) is called **cancelling**.

Problem 5. Evaluate: $1\frac{3}{5}\times 2\frac{1}{3}\times 3\frac{3}{7}$

Mixed numbers **must** be expressed as improper fractions before multiplication can be performed. Thus,

$$1\frac{3}{5}\times 2\frac{1}{3}\times 3\frac{3}{7}$$
$$=\left(\frac{5}{5}+\frac{3}{5}\right)\times\left(\frac{6}{3}+\frac{1}{3}\right)\times\left(\frac{21}{7}+\frac{3}{7}\right)$$

$$= \frac{8}{5} \times \frac{{}^1\cancel{7}}{{}_1\cancel{3}} \times \frac{\cancel{24}^{\,8}}{\cancel{7}_{\,1}} = \frac{8 \times 1 \times 8}{5 \times 1 \times 1}$$

$$= \frac{64}{5} = \mathbf{12\frac{4}{5}}$$

Problem 6. Simplify: $\frac{3}{7} \div \frac{12}{21}$

$$\frac{3}{7} \div \frac{12}{21} = \frac{\frac{3}{7}}{\frac{12}{21}}$$

Multiplying both numerator and denominator by the reciprocal of the denominator gives:

$$\frac{\frac{3}{7}}{\frac{12}{21}} = \frac{\frac{{}^1\cancel{3}}{{}_1\cancel{7}} \times \frac{\cancel{21}^{\,3}}{\cancel{12}_{\,4}}}{\frac{{}^1\cancel{12}}{{}_1\cancel{21}} \times \frac{\cancel{21}^{\,1}}{\cancel{12}_{\,1}}} = \frac{\frac{3}{4}}{1} = \mathbf{\frac{3}{4}}$$

This method can be remembered by the rule: invert the second fraction and change the operation from division to multiplication. Thus:

$$\frac{3}{7} \div \frac{12}{21} = \frac{{}^1\cancel{3}}{{}_1\cancel{7}} \times \frac{\cancel{21}^{\,3}}{\cancel{12}_{\,4}} = \mathbf{\frac{3}{4}} \text{ as obtained previously.}$$

Problem 7. Find the value of $5\frac{3}{5} \div 7\frac{1}{3}$

The mixed numbers must be expressed as improper fractions. Thus,

$$5\frac{3}{5} \div 7\frac{1}{3} = \frac{28}{5} \div \frac{22}{3} = \frac{{}^{14}\cancel{28}}{5} \times \frac{3}{\cancel{22}_{\,11}} = \mathbf{\frac{42}{55}}$$

Problem 8. Simplify:

$$\frac{1}{3} - \left(\frac{2}{5} + \frac{1}{4}\right) \div \left(\frac{3}{8} \times \frac{1}{3}\right)$$

The order of precedence of operations for problems containing fractions is the same as that for integers, i.e. remembered by **BODMAS** (**B**rackets, **O**f, **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction). Thus,

$$\frac{1}{3} - \left(\frac{2}{5} + \frac{1}{4}\right) \div \left(\frac{3}{8} \times \frac{1}{3}\right)$$

$$= \frac{1}{3} - \frac{4 \times 2 + 5 \times 1}{20} \div \frac{\cancel{3}^{\,1}}{\cancel{24}_{\,8}} \qquad \text{(B)}$$

$$= \frac{1}{3} - \frac{13}{{}_5\cancel{20}} \times \frac{\cancel{8}^{\,2}}{1} \qquad \text{(D)}$$

$$= \frac{1}{3} - \frac{26}{5} \qquad \text{(M)}$$

$$= \frac{(5 \times 1) - (3 \times 26)}{15} \qquad \text{(S)}$$

$$= \frac{-73}{15} = \mathbf{-4\frac{13}{15}}$$

Problem 9. Determine the value of

$$\frac{7}{6} \text{ of } \left(3\frac{1}{2} - 2\frac{1}{4}\right) + 5\frac{1}{8} \div \frac{3}{16} - \frac{1}{2}$$

$$\frac{7}{6} \text{ of } \left(3\frac{1}{2} - 2\frac{1}{4}\right) + 5\frac{1}{8} \div \frac{3}{16} - \frac{1}{2}$$

$$= \frac{7}{6} \text{ of } 1\frac{1}{4} + \frac{41}{8} \div \frac{3}{16} - \frac{1}{2} \qquad \text{(B)}$$

$$= \frac{7}{6} \times \frac{5}{4} + \frac{41}{8} \div \frac{3}{16} - \frac{1}{2} \qquad \text{(O)}$$

$$= \frac{7}{6} \times \frac{5}{4} + \frac{41}{{}_1\cancel{8}} \times \frac{\cancel{16}^{\,2}}{3} - \frac{1}{2} \qquad \text{(D)}$$

$$= \frac{35}{24} + \frac{82}{3} - \frac{1}{2} \qquad \text{(M)}$$

$$= \frac{35 + 656}{24} - \frac{1}{2} \qquad \text{(A)}$$

$$= \frac{691}{24} - \frac{1}{2} \qquad \text{(A)}$$

$$= \frac{691 - 12}{24} \qquad \text{(S)}$$

$$= \frac{679}{24} = \mathbf{28\frac{7}{24}}$$

Now try the following Practice Exercise

Practice Exercise 1 Fractions (Answers on page 656)

Evaluate the following:

1. (a) $\frac{1}{2} + \frac{2}{5}$ (b) $\frac{7}{16} - \frac{1}{4}$

2. (a) $\frac{2}{7}+\frac{3}{11}$ (b) $\frac{2}{9}-\frac{1}{7}+\frac{2}{3}$

3. (a) $10\frac{3}{7}-8\frac{2}{3}$ (b) $3\frac{1}{4}-4\frac{4}{5}+1\frac{5}{6}$

4. (a) $\frac{3}{4}\times\frac{5}{9}$ (b) $\frac{17}{35}\times\frac{15}{119}$

5. (a) $\frac{3}{5}\times\frac{7}{9}\times1\frac{2}{7}$ (b) $\frac{13}{17}\times4\frac{7}{11}\times3\frac{4}{39}$

6. (a) $\frac{3}{8}\div\frac{45}{64}$ (b) $1\frac{1}{3}\div2\frac{5}{9}$

7. $\frac{1}{2}+\frac{3}{5}\div\frac{8}{15}-\frac{1}{3}$

8. $\frac{7}{15}$ of $\left(15\times\frac{5}{7}\right)+\left(\frac{3}{4}\div\frac{15}{16}\right)$

9. $\frac{1}{4}\times\frac{2}{3}-\frac{1}{3}\div\frac{3}{5}+\frac{2}{7}$

10. $\left(\frac{2}{3}\times1\frac{1}{4}\right)\div\left(\frac{2}{3}+\frac{1}{4}\right)+1\frac{3}{5}$

11. If a storage tank is holding 450 litres when it is three-quarters full, how much will it contain when it is two-thirds full?

12. Three people, P, Q and R contribute to a fund. P provides 3/5 of the total, Q provides 2/3 of the remainder, and R provides £8. Determine (a) the total of the fund, (b) the contributions of P and Q.

1.2 Ratio and proportion

The ratio of one quantity to another is a fraction, and is the number of times one quantity is contained in another quantity **of the same kind**. If one quantity is **directly proportional** to another, then as one quantity doubles, the other quantity also doubles. When a quantity is **inversely proportional** to another, then as one quantity doubles, the other quantity is halved.

Problem 10. A piece of timber 273 cm long is cut into three pieces in the ratio of 3 to 7 to 11. Determine the lengths of the three pieces

The total number of parts is $3+7+11$, that is, 21. Hence 21 parts correspond to 273 cm

1 part corresponds to $\frac{273}{21}=13$ cm

3 parts correspond to $3\times13=39$ cm

7 parts correspond to $7\times13=91$ cm

11 parts correspond to $11\times13=143$ cm

i.e. **the lengths of the three pieces are 39 cm, 91 cm and 143 cm**.

(Check: $39+91+143=273$)

Problem 11. A gear wheel having 80 teeth is in mesh with a 25 tooth gear. What is the gear ratio?

$$\text{Gear ratio}=80:25=\frac{80}{25}=\frac{16}{5}=3.2$$

i.e. gear ratio = **16 : 5** or **3.2 : 1**

Problem 12. An alloy is made up of metals A and B in the ratio 2.5 : 1 by mass. How much of A has to be added to 6 kg of B to make the alloy?

Ratio A : B: :2.5 : 1 (i.e. A is to B as 2.5 is to 1) or $\frac{A}{B}=\frac{2.5}{1}=2.5$

When B = 6 kg, $\frac{A}{6}=2.5$ from which,

A $=6\times2.5=$ **15 kg**

Problem 13. If 3 people can complete a task in 4 hours, how long will it take 5 people to complete the same task, assuming the rate of work remains constant?

The more the number of people, the more quickly the task is done, hence inverse proportion exists.

3 people complete the task in 4 hours.
1 person takes three times as long, i.e.

$4\times3=12$ hours,

5 people can do it in one fifth of the time that one person takes, that is $\frac{12}{5}$ hours or **2 hours 24 minutes**.

Now try the following Practice Exercise

Practice Exercise 2 Ratio and proportion (Answers on page 656)

1. Divide 621 cm in the ratio of 3 to 7 to 13.
2. When mixing a quantity of paints, dyes of four different colours are used in the ratio of 7 : 3 : 19 : 5. If the mass of the first dye used is $3\frac{1}{2}$ g, determine the total mass of the dyes used.
3. Determine how much copper and how much zinc is needed to make a 99 kg brass ingot if they have to be in the proportions copper : zinc: :8 : 3 by mass.
4. It takes 21 hours for 12 men to resurface a stretch of road. Find how many men it takes to resurface a similar stretch of road in 50 hours 24 minutes, assuming the work rate remains constant.
5. It takes 3 hours 15 minutes to fly from city A to city B at a constant speed. Find how long the journey takes if
   (a) the speed is $1\frac{1}{2}$ times that of the original speed and
   (b) if the speed is three-quarters of the original speed.

1.3 Decimals

The decimal system of numbers is based on the **digits** 0 to 9. A number such as 53.17 is called a **decimal fraction**, a decimal point separating the integer part, i.e. 53, from the fractional part, i.e. 0.17

A number which can be expressed exactly as a decimal fraction is called a **terminating decimal** and those which cannot be expressed exactly as a decimal fraction are called **non-terminating decimals**. Thus, $\frac{3}{2} = 1.5$ is a terminating decimal, but $\frac{4}{3} = 1.33333\ldots$ is a non-terminating decimal. 1.33333... can be written as $1.\dot{3}$, called 'one point-three recurring'.

The answer to a non-terminating decimal may be expressed in two ways, depending on the accuracy required:

(i) correct to a number of **significant figures**, that is, figures which signify something, and

(ii) correct to a number of **decimal places**, that is, the number of figures after the decimal point.

The last digit in the answer is unaltered if the next digit on the right is in the group of numbers 0, 1, 2, 3 or 4, but is increased by 1 if the next digit on the right is in the group of numbers 5, 6, 7, 8 or 9. Thus the non-terminating decimal 7.6183... becomes 7.62, correct to 3 significant figures, since the next digit on the right is 8, which is in the group of numbers 5, 6, 7, 8 or 9. Also 7.6183... becomes 7.618, correct to 3 decimal places, since the next digit on the right is 3, which is in the group of numbers 0, 1, 2, 3 or 4

Problem 14. Evaluate: $42.7 + 3.04 + 8.7 + 0.06$

The numbers are written so that the decimal points are under each other. Each column is added, starting from the right.

$$\begin{array}{r} 42.7 \\ 3.04 \\ 8.7 \\ 0.06 \\ \hline 54.50 \\ \hline \end{array}$$

Thus $\mathbf{42.7 + 3.04 + 8.7 + 0.06 = 54.50}$

Problem 15. Take 81.70 from 87.23

The numbers are written with the decimal points under each other.

$$\begin{array}{r} 87.23 \\ -81.70 \\ \hline 5.53 \\ \hline \end{array}$$

Thus $\mathbf{87.23 - 81.70 = 5.53}$

Problem 16. Find the value of

$$23.4 - 17.83 - 57.6 + 32.68$$

The sum of the positive decimal fractions is

$$23.4 + 32.68 = 56.08$$

The sum of the negative decimal fractions is

$$17.83 + 57.6 = 75.43$$

Taking the sum of the negative decimal fractions from the sum of the positive decimal fractions gives:

$$56.08 - 75.43$$

i.e. $-(75.43 - 56.08) = \mathbf{-19.35}$

Problem 17. Determine the value of 74.3×3.8

When multiplying decimal fractions: (i) the numbers are multiplied as if they are integers, and (ii) the position of the decimal point in the answer is such that there are as many digits to the right of it as the sum of the digits to the right of the decimal points of the two numbers being multiplied together. Thus

(i)
$$\begin{array}{r} 743 \\ 38 \\ \hline 5\,944 \\ 22\,290 \\ \hline 28\,234 \\ \hline \end{array}$$

(ii) As there are $(1+1)=2$ digits to the right of the decimal points of the two numbers being multiplied together, $(74.\underline{3} \times 3.\underline{8})$, then

$$\mathbf{74.3 \times 3.8 = 282.34}$$

Problem 18. Evaluate $37.81 \div 1.7$, correct to (i) 4 significant figures and (ii) 4 decimal places

$$37.81 \div 1.7 = \frac{37.81}{1.7}$$

The denominator is changed into an integer by multiplying by 10. The numerator is also multiplied by 10 to keep the fraction the same. Thus

$$37.81 \div 1.7 = \frac{37.81 \times 10}{1.7 \times 10} = \frac{378.1}{17}$$

The long division is similar to the long division of integers and the first four steps are as shown:

$$\begin{array}{r} 22.24117.. \\ 17\,\overline{)\;378.100000} \\ \underline{34} \\ 38 \\ \underline{34} \\ 41 \\ \underline{34} \\ 70 \\ \underline{68} \\ 20 \end{array}$$

(i) **$37.81 \div 1.7 = 22.24$, correct to 4 significant figures**, and

(ii) **$37.81 \div 1.7 = 22.2412$, correct to 4 decimal places.**

Problem 19. Convert (a) 0.4375 to a proper fraction and (b) 4.285 to a mixed number

(a) 0.4375 can be written as $\dfrac{0.4375 \times 10\,000}{10\,000}$ without changing its value,

i.e. $0.4375 = \dfrac{4375}{10\,000}$

By cancelling

$$\frac{4375}{10\,000} = \frac{875}{2000} = \frac{175}{400} = \frac{35}{80} = \frac{7}{16}$$

i.e. $\mathbf{0.4375 = \dfrac{7}{16}}$

(b) Similarly, $\mathbf{4.285} = 4\dfrac{285}{1000} = \mathbf{4\dfrac{57}{200}}$

Problem 20. Express as decimal fractions:

(a) $\dfrac{9}{16}$ and (b) $5\dfrac{7}{8}$

(a) To convert a proper fraction to a decimal fraction, the numerator is divided by the denominator. Division by 16 can be done by the long division method, or, more simply, by dividing by 2 and then 8:

$$2\,\overline{)\,9.00}^{\;4.50} \qquad 8\,\overline{)\,4.5000}^{\;0.5625}$$

Thus $\mathbf{\dfrac{9}{16} = 0.5625}$

(b) For mixed numbers, it is only necessary to convert the proper fraction part of the mixed number to a decimal fraction. Thus, dealing with the $\frac{7}{8}$ gives:

$$8\,\overline{)\,7.000}^{\;0.875} \qquad \text{i.e. } \frac{7}{8} = 0.875$$

Thus $\mathbf{5\dfrac{7}{8} = 5.875}$

Now try the following Practice Exercise

Practice Exercise 3 Decimals (Answers on page 656)

In Problems 1 to 6, determine the values of the expressions given:

1. $23.6 + 14.71 - 18.9 - 7.421$
2. $73.84 - 113.247 + 8.21 - 0.068$

3. $3.8 \times 4.1 \times 0.7$

4. 374.1×0.006

5. $421.8 \div 17$, (a) correct to 4 significant figures and (b) correct to 3 decimal places.

6. $\frac{0.0147}{2.3}$, (a) correct to 5 decimal places and (b) correct to 2 significant figures.

7. Convert to proper fractions:
(a) 0.65 (b) 0.84 (c) 0.0125 (d) 0.282 and (e) 0.024

8. Convert to mixed numbers:
(a) 1.82 (b) 4.275 (c) 14.125 (d) 15.35 and (e) 16.2125

In Problems 9 to 12, express as decimal fractions to the accuracy stated:

9. $\frac{4}{9}$, correct to 5 significant figures.

10. $\frac{17}{27}$, correct to 5 decimal places.

11. $1\frac{9}{16}$, correct to 4 significant figures.

12. $13\frac{31}{37}$, correct to 2 decimal places.

13. Determine the dimension marked x in the length of shaft shown in Figure 1.1. The dimensions are in millimetres.

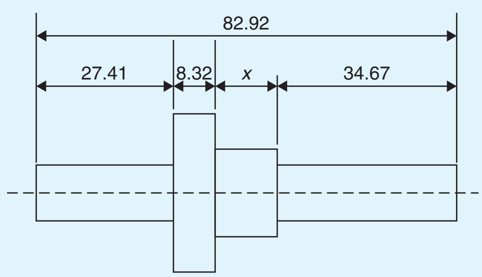

Figure 1.1

14. A tank contains 1800 litres of oil. How many tins containing 0.75 litres can be filled from this tank?

1.4 Percentages

Percentages are used to give a common standard and are fractions having the number 100 as their denominators. For example, 25 per cent means $\frac{25}{100}$ i.e. $\frac{1}{4}$ and is written 25%

Problem 21. Express as percentages:
(a) 1.875 and (b) 0.0125

A decimal fraction is converted to a percentage by multiplying by 100. Thus,

(a) 1.875 corresponds to $1.875 \times 100\%$, i.e. **187.5%**

(b) 0.0125 corresponds to $0.0125 \times 100\%$, i.e. **1.25%**

Problem 22. Express as percentages:

(a) $\frac{5}{16}$ and (b) $1\frac{2}{5}$

To convert fractions to percentages, they are (i) converted to decimal fractions and (ii) multiplied by 100

(a) By division, $\frac{5}{16} = 0.3125$, hence $\frac{5}{16}$ corresponds to $0.3125 \times 100\%$, i.e. **31.25%**

(b) Similarly, $1\frac{2}{5} = 1.4$ when expressed as a decimal fraction.

Hence $1\frac{2}{5} = 1.4 \times 100\% = \mathbf{140\%}$

Problem 23. It takes 50 minutes to machine a certain part, Using a new type of tool, the time can be reduced by 15%. Calculate the new time taken

$$15\% \text{ of } 50 \text{ minutes} = \frac{15}{100} \times 50 = \frac{750}{100}$$
$$= 7.5 \text{ minutes.}$$

hence the **new time taken** is

$$50 - 7.5 = \mathbf{42.5 \text{ minutes}}.$$

Alternatively, if the time is reduced by 15%, then it now takes 85% of the original time, i.e. 85% of $50 = \frac{85}{100} \times 50 = \frac{4250}{100} = \mathbf{42.5 \text{ minutes}}$, as above.

Problem 24. Find 12.5% of £378

12.5% of £378 means $\frac{12.5}{100} \times 378$, since per cent means 'per hundred'.